## ALGEBRA IN THE ELEMENTARY SCHOOLS.

By James H. Shipley.

I have never yet heard of a high school teacher's being satisfied with the first-year pupil's knowledge of common fractions, and the other day a I. A. girl frankly admitted that she could not multiply $12\frac{2}{3}$ by $15\frac{3}{4}$ because she had skipped one grade, had then had algebra and geometry, and the review didn't touch on mixed numbers. Of course, this was not the fault of any teacher, but the fault of a system which is trying to crowd too many things into too small a space. The real efficiency of the schools does not depend upon their being housed in million-dollar buildings, under a well-organized administrative force, excellent equipment, etc.; it depends upon two things,—what is taught, and how it is taught; and more especially upon the latter; for every teacher of mathematics knows that a pupil can derive as much permanent good from the study of a very few topics or theorems, so presented as to be pleasing to the pupil, or at least interesting, and at the same time make him think, as he can from ten times the amount of material "rammed home" with the sole object of being reproduced at examinations. The policy of standardizing everything by examinations is doing our expensive school system an untold injury; the report of the city superintendent compares the schools according to the number of their pupils who pass the examinations, and the principal warns the teacher that he is rated according to the number of his pupils that pass, and this pressure is passed on to the pupil. Until some method of close class-room observation and supervision is introduced with a view to allowing free rein to a teacher's individuality and originality even at the expense of his pupil's failing the conventional examinations, it is hardly worth while suggesting other changes.

And *what* is taught is not of paramount importance; a well-informed, broad-minded, ingenious teacher of history, who can instill patriotism into a pupil and impart to him a desire for further reading of history after he leaves school, can do him

more good, as a future citizen, than a poor teacher of algebra might; or a teacher of algebra, by presenting the subject in an intelligent and intelligible way so as not to overburden the pupil or put him under too much duress, but lead him to see some of the interesting features of the subject, can do him more good than a poor teacher of history, who only tries to make the pupil accumulate facts. And so with the mathematics in the elementary schools; if all teachers are obliged to cover more matter than they really have time for, no one can do anyone any good, and poor teachers and good teachers are reduced to about the same level. When a person stops to think of all the things that he has learned at one time or another, and then considers how much of it he remembers, he can realize of how little importance the subject matter really is. Of two boys, taught in two different ways, for two different reasons, one might say that " $\pi$ is the ratio of the circumference of a circle to its diameter," and the other would say that he had measured the circumferences and diameters of several circles and always found the circumference to be a little more than three times as long as the diameter.

As many things as possible should be forced out of the elementary curriculum so that what remained might be studied more leisurely and thoroughly. For example, to try to teach music to some of the boys is a farce, for the boy " who hath not music in his soul " is more " fit for rapine, plunder, and murder " after the process than before; and about the same can be said of free-hand drawing and several other things. In place of these and geometry, could and should be introduced a course in elementary mechanical draughting. This is something in which 99 boys out of a 100 are interested; it teaches them to be neat and accurate, it interests them in geometric forms and prepares the way for formal geometry, and it is the beginning of a profession; for a good draughtsman is usually in demand.

As for pupils getting algebra in the elementary schools, there is no reason why they should not have it from the very start, and become as used to the algebraic methods as they are to the arithmetic, for the algebraic method is clearer and admits more readily of explanation; it would also admit a larger variety of verbal problems, even to the extent of two unknown quantities, which pupils find very easy. The fact of these problems being practical or not is immaterial—the effort to introduce so called

practical things into the curriculum is somewhat misdirected, for if a pupil in either the elementary or high schools can be taught enough English and arithmetic to see him through life, and in addition can be taught to think quickly and correctly and independently, he has acquired the most practical thing possible, and it makes little difference what he has studied to acquire this end, so long as he has been given an impulse to continue his work in literature or history or language or mathematics or biology, after he leaves school, as a recreation or pastime, free from compunction.

There are many ways in which algebraic methods can be used in arithmetic with the result of making processes appear more rational, thereby breaking up the mechanical method of presenting matter, and at the same time preparing for formal algebra those who are destined to take it. For example, pupils always seem to be interested in discovering a new way of doing the following example, $7\cdot5+6\cdot3+6\cdot7+8\cdot3+9\cdot7+3\cdot11$, that is, instead of finding the value of each term in order, and adding, taking them by 7's and 3's, $5\cdot7+6\cdot7+9\cdot7=20\cdot7=140$; and $6\cdot3+8\cdot3+11\cdot3=25\cdot3=75$, and $140+75=215$. This is an important algebraic principle which many authors even have overlooked, and pupils seem to like it; it can be used in addition and subtraction:

$$\begin{array}{r} 11\cdot7 \\ 3\cdot7 \\ \underline{15\cdot7} \\ 29\cdot7 \\ 203 \end{array} \qquad\qquad \begin{array}{r} 8\tfrac{1}{2}\cdot13 \\ 3\ \ \cdot13 \\ \hline 5\tfrac{1}{2}\cdot13 \\ 71\tfrac{1}{2} \end{array}$$

Few pupils coming from the grammar schools can add a long column in groups of 10 which is so much more rapid and which helps them in algebra; and the majority of them are quite surprised when told that the easiest way to subtract is to add, very much like cashiers do. This idea is very helpful both in arithmetic and algebra, for example,

$$\begin{array}{r} 15\tfrac{2}{3} \\ \underline{6\tfrac{3}{4}} \\ 8\tfrac{11}{12} \end{array} \qquad\qquad \begin{array}{r} 8\tfrac{1}{2} \\ \underline{-\tfrac{3}{4}} \\ 7\tfrac{3}{4} \end{array}$$

In order to get $15\tfrac{2}{3}$ it is necessary to add to the subtrahend

$$8 + \frac{1}{4} + \frac{2}{3} = 8\frac{11}{12}.$$

In the second example it is necessary to add 7 and $\frac{1}{4}$ and $\frac{1}{2}$ to $\frac{3}{4}$ in order to get $8\frac{1}{2}$.

Multiplication can be easily explained and many examples made easier by the algebraic method. How many pupils in multiplying 47 by 5 would multiply the 40 first and then the 7, or how many really know that it is $40 + 7$ multiplied by 5, and it makes no difference which one is multiplied first; or that the quickest way to multiply 723 by 3 is to consider it as $700 + 20 + 3$ and multiply from the left. This same principle appears in denominate numbers, and this subject can be taught more rationally in this way, instead of by rule—I think that it is a mistake to ever introduce a rule.

$$\begin{array}{r} 5 \text{ yds.} + 2 \text{ ft.} + 8 \text{ in.} \\ 4 \\ \hline 20 \text{ yds.} + 8 \text{ ft.} + 32 \text{ in.} \end{array}$$

Each one must be multiplied, and then the pupil can change these denominations afterwards.

In multiplying 45 by 37 what we really do is to multiply the 40 and 5 first by 7 and then by 30 and add, but it can be done mentally very easily by beginning with 30, thus

$$\begin{array}{r} 40 + 5 \\ 30 + 7 \\ \hline 1200 \\ 150 \\ 280 \\ 35 \\ \hline 1665 \end{array}$$

To square a number by this principle is still easier as $(50 + 7)^2$.

Such a presentation of multiplication has three advantages, it shows the pupil a reason for the usual arrangement of numbers in multiplication, it gives him an easy way of doing examples mentally in less time than by writing, and it prepares him for the general principle of $(a + b)(x + y)$ if he goes on to algebra. This same principle is of great use in fractions, yet not one pupil in 75 can apply it to such an example as $12\frac{2}{3} \cdot 15\frac{3}{4}$, and I doubt if all teachers can, but why should not this method

be generally used? In the first place, how many pupils have had it called to their attention that $12\frac{2}{3}$ means $12 + \frac{2}{3}$? Mistakes are continually made in surds on account of this very thing; for example $\frac{2}{3}\sqrt{12} = 2\frac{2}{3}\sqrt{3}$. But to come back to $12\frac{2}{3} \cdot 15\frac{3}{4}$, which nearly all pupils would do by reducing to fractions, how easy a mental problem it becomes if taken as

$$\begin{array}{c} 12 + \frac{2}{3} \\ \underline{15 + \frac{3}{4}} \end{array}$$

The same is true of division, such as $12\frac{3}{4} \div 3$ or $27\frac{3}{4}$ by 4, and 125 yds. + 2 ft. + 11 in. divided by 3—there would be $\frac{1}{3}$ as many yds., $\frac{1}{3}$ as many ft., and $\frac{1}{3}$ as many inches, and then these could be changed afterwards. In an ordinary example like $2316 \div 24 = 96\frac{1}{2}$ (remainder 12) there is hardly a pupil who could not get the correct answer, yet hardly one who could tell *what* he really does with the remainder—he doesn't realize that he has been dividing the dividend by parts, and that the part remaining must also be divided by 24, giving $\frac{12}{24}$ or $\frac{1}{2}$, which must be *added* to the quotient as a part of it; from not knowing this, beginners in algebra get such quotients as $a - b\frac{b}{a+b}$.

The algebraic method of finding the H. C. F. and L. C. M. recommends itself on the same grounds as these other principles, that is, it makes the work easier, lends itself more readily to explanation, and is therefore more rational and less mechanical; $3 \cdot 5 \cdot 7 \cdot 2$ can more readily be divided by $2 \cdot 7$ than 210 can by 14, and they are the same thing in two different forms. So in the following example, the algebraic method can be used throughout

$$\frac{11}{21} + \frac{9}{10} + \frac{7}{15} + \frac{13}{14} =$$

$$3 \cdot 7 \quad 2 \cdot 5 \quad 3 \cdot 5 \quad 2 \cdot 7$$

$$\frac{11 \cdot 2 \cdot 5 + 9 \cdot 3 \cdot 7 + 7 \cdot 2 \cdot 7 + 13 \cdot 3 \cdot 5}{2 \cdot 3 \cdot 5 \cdot 7} = \frac{110 + 189 + 98 + 195}{210} = \text{etc.}$$

Fractions afford the greatest field for algebraic work. Cancellation as such should be abolished; pupils use it on all occasions, everywhere, and are usually wrong; cancellation seems to mean to them drawing lines through any two things that happen

to look alike. A great deal of such work can be prevented by keeping away from rules, and giving axioms as reasons for doing things. Often on asking a pupil what he has done and why he has done it, he points helplessly at his correct work and lets it speak for itself. For example instead of teaching, as is usual, that $\frac{2}{3}$ is changed to 12ths by dividing 12 by 3 and multiplying by 2, an axiom should be introduced to the effect that we can multiply the numerator and denominator of a fraction by the same number and not change its value, and then the only question is "by what is it necessary to multiply?" The same is true in reducing fractions to lower terms—if we make the parts 3 times as large we need only $\frac{1}{3}$ as many, etc. Pupils like to see this compared with adding the same number to numerator and denominator. When a pupil has seen these axioms he has a reason for doing things, and he also has something useful if he is going on to algebra proper.

In arithmetic there could be a more extended use of algebraic symbols, especially parentheses. Who knows what $30 \div 5 \times 2$ means unless some one has told him? If two successive examples could be written $(30 \div 5) \times 2$ and $30 \div (5 \times 2)$ the pupil would be made to think, and anything tending to this end is a "consummation devoutly to be wished"; also, is there any difference between $10 - 7 + 2$ and $10 - (7 + 2)$? Instead of the cumbersome sign of multiplication, the dot could be used as in algebra. Complex fractions, in which are involved so much patience and care, could be extended, and attention drawn to such combinations as $\frac{2/5}{3}$ and $\frac{2}{5/3}$, etc.

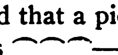
Arithmetic and geometric progressions could be introduced in an elementary way as an interesting side light on insurance and interest, as well as in other ways, and even very young children could get lots of pleasure out of combinations and permutations if adroitly handled—give them pieces of card-board in four different colors and let them see in how many different ways they could arrange the four, or how many different groups of two they could get; or let the boys figure out how many different base-ball batteries they could make up from their old arithmetic friends John, James, Henry, and Wm. as catchers, and Reginald, Percy, Fortescue, and Mike as pitchers; and after they have found out a few of these things, tell them that the seven boys in

the first row could be seated in 5,040 different ways, or that the captain of an eight-oared crew has a choice of about 40,000 different ways of arranging his men, or that the ten books on the teacher's desk might easily get out of order, as there are 3,600,-000 different orders that they could be in. A grilling and gruelling recitation in algebra or arithmetic might very profitably be stopped by a gentle transition into a discussion of the various methods of signalling on a warship, or into a lesson on word-analysis which is so sadly lacking from all of our courses—how many teachers have time to call attention to the endings of addend, minuend, dividend, multiplicand, or show that a dividend is so called for the same reason as some of the things that the Standard Oil declares; and would it not be true that pupils would be less apt to confound subtrahend and minuend if they knew that the former came from "sub" and "traho, or tractum," and for the same reason, less prone to say that the product of 3 and 5 is 8 as some invariably do in every class that comes into the high school? If a pupil is asked to go down one certain stairway out of several, he is more likely to remember which one to take if he knows the reason for going that way. It is such side-stepping from the regular hum-drum work of school that gives the pupils some interest in their work, and broadens their view, and a teacher who can pause long enough in the mad rush for examinations to present some such alleviating information in an interesting talk is really a "superior teacher" and is doing the pupil more good than one who can force ten times the amount of *knowledge* into the pupil's head with the result of disgusting him with the subject, the teacher, and the school; *such* "superior" teachers, and there are many of them, should be sought out and besought to radiate their effulgence on their less "superior" comrades.

Just one more topic, and that is graphs. As used in elementary algebra they are simply a kindergarten method of representing only crudely and more or less clearly, usually less, something which is already as clear as it can be made; in algebra

$$2x + 3y = 5$$
$$3x + 2y = 7$$

means that there are in existence two numbers such that two times one of them plus three times the other equals five, and

three times the first plus two times the second equals seven; and then there is a clear way of finding what those numbers are. To invent an entirely new system in order to put a new interpretation upon what is already clear, seems unnecessary; yet it is interesting, and should by all means serve as a side issue to be followed up by an elementary discussion of curves; for any pupil in either the elementary or high school is interested in learning how to make an ellipse with a cord, and to find that it is really only a flattened circle with two centers instead of one, and that the sun shining through a round hole in the shade casts an ellipse on the wall and that the curtain cord forms a catenary, and that a piece of tape on the tire of a moving bicycle goes like this —and dozens of other things which make the pupils observing. But the real place for graphic tables and curves of all kinds is arithmetic; here is a great field for the tabulation of all sorts of interesting features and statistics—the boys could arrange a table for the base-ball league or interest tables for varying sums, time, and rates, tables of railroad fares, multiplication tables, etc. An ingenious teacher could make such work tremendously helpful and not boresome, and in connection only with a versatile teacher of English could give a boy a more useful education than he is getting now—more useful because it could be made more reasoning and resourceful.

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